Until now: (post-)Newtonian approximation Ehlers 2017 L3P1

slow motion & weak field

$$\frac{V^2}{c^2} \sim \frac{GM}{rc^2} \sim \varepsilon_{PN}$$

breaks down when ε_{PN} approaches 1!

Dynamics in strong G-fields

Methods: - numerical simulations → lectures Harald Pfeiffer

- test-mass in black-hole (BH) spacetime → here

$\mu \ll M$, $M = m_1 + m_2$, $\mu = \frac{m_1 m_2}{M}$

non-rot. BH (simple):

$$d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu$$

$$= \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

4 Killing vectors ξ_ν with $\nabla_{(\mu} \xi_{\nu)} = 0$

⇒ 4 conserved quantities $\xi_\mu u^\mu = \text{const}$

Energy, angular momentum

case $(\xi^\mu) = (1,0,0,0)$: $E = -u_\mu \xi^\mu = -u_0 = \frac{\text{energy}}{\text{mass } \mu}$ is conserved

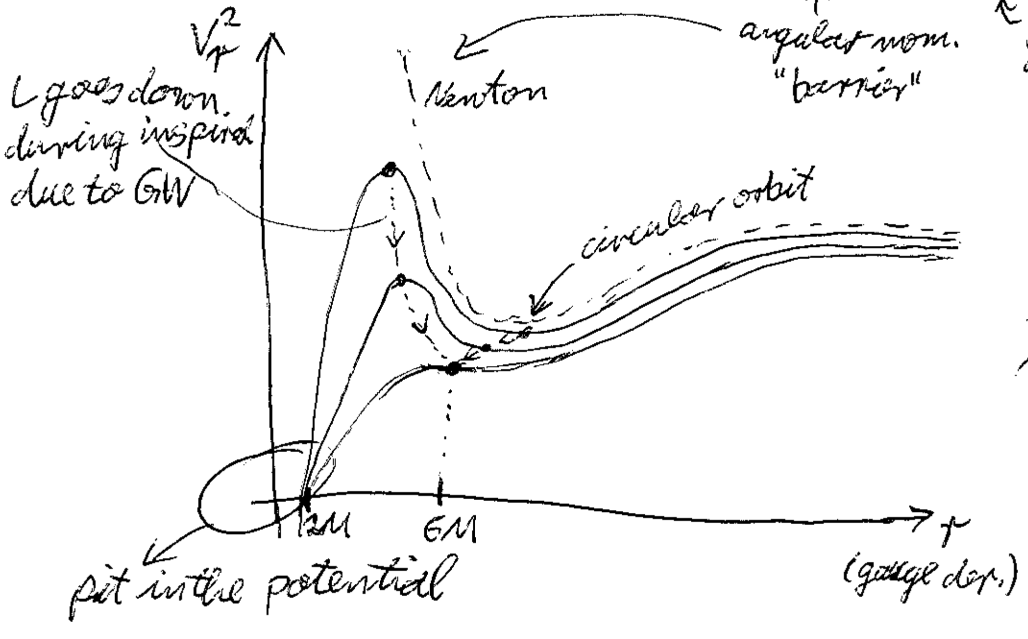
case $(\xi^\mu) = (0,0,0,1)$: $L = u_\mu \xi^\mu = u_3 = \frac{\text{ang. mom.}}{\mu}$ " } (*)

use (*) in $g^{\mu\nu} u_\mu u_\nu = -1$ & equatorial plane $(\theta = \frac{\pi}{2}, u^\theta = 0)$

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 - V_r^2, \quad V_r^2 = \left(1 + \frac{L^2}{r^2}\right)\left(1 - \frac{2M}{r}\right)$$

effective radial potential

(↑ angular mom. "barrier"; ↑ grav. attraction)



min and max move together during inspiral

⇒ they form a saddle point

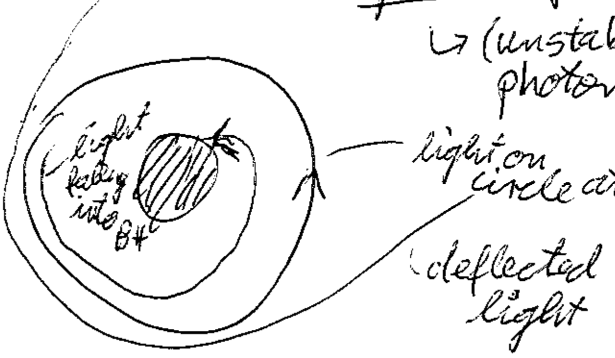
⇒ no stable orbit any more for $r \leq 6M$

⇒ last stable circular orbit (LSCO)

⇒ test-mass plunges into BH

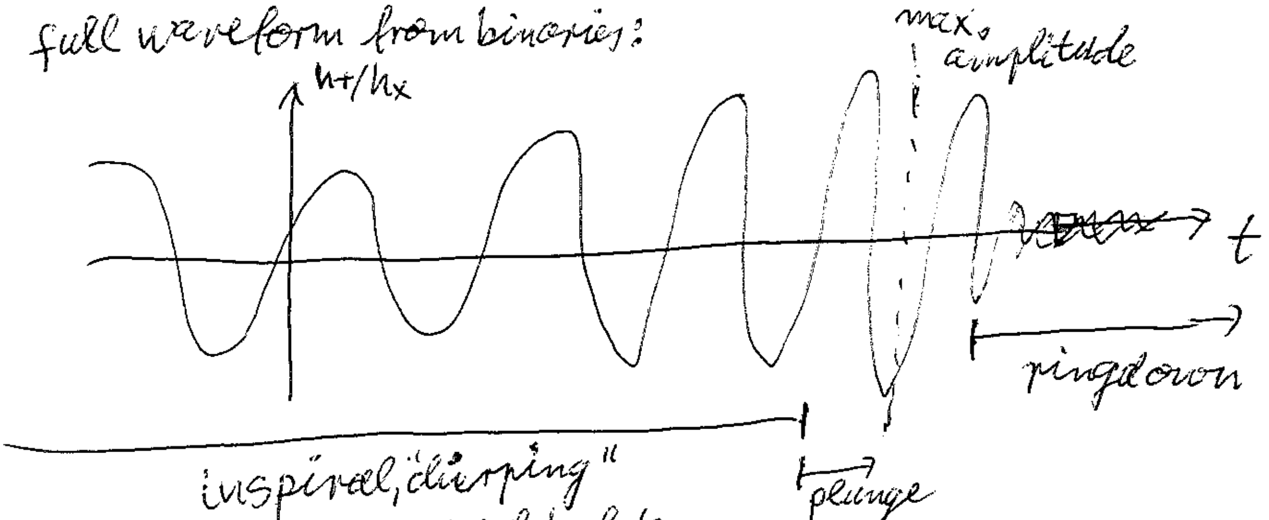
gravity wins!

another strong-field feature:
<u>light-ring</u>
↳ (unstable) circular photon orbit

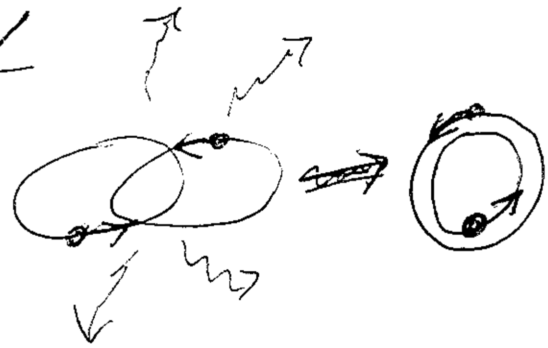


ringdown of final/merged black hole: GW resonate within light ring, frequency~M

full waveform from binaries:

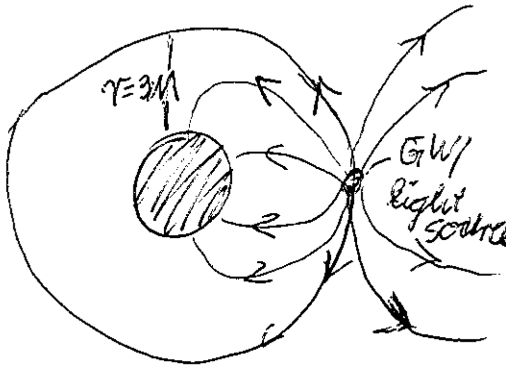


Phases of binary ~~inspiral systems~~ black hole's GW



- early inspiral:
  elliptic orbit becomes almost circular due to emission of GW
  "dissipation brings system to a (quasi-)equilibrium"

- late inspiral ≈ circular, chirp
  amp. ↗ ω ↗ until LSCO r~6M
- LSCO: gravity wins ⇝ plunge
- light ring, r~3M
  GW start to fall into the other BH
  ≈ half of the GW fall in
  ≈ point of max. amplitude



- common event horizon forms, r~2M
  ↳ <u>merger</u> 8

- Ringdown of final deformed BH Ehlers 2017 L3P3

GW resonate within light-ring of final BH

⇝ frequency estimate (nonrot. BH):

$$\omega_{\text{ringdown}} \approx 2\omega \approx \frac{2}{M}\left(\frac{M}{r}\right)^{3/2} \approx \frac{2}{3\sqrt{3}M}$$

(↑ 3rd Kepler; ↑ $r \approx 3M$)

but: final BH usually spinning fast (recoil)

inspiral ⇝ M_c

max. amplitude, ringdown } ⇝ M

} ⇝ individual masses from full inspiral-merger-ringdown (IMR) waveforms

Waveform models

PN waveforms: inspiral only → not enough!

Phenom waveforms: PN inspiral, fit of plunge/merger/ringdown to numerical relativity

Effective one body waveforms: developed in our division

combine PN (valid for inspiral) with test-mass motion, attach ringdown at light-ring

↳ prediction for IMR waveform then calibrate model to match numerical relativity

↳ synergy of all methods

but: line between fit and calibration can be thin.

basic idea: Start with one-body motion ~ mass μ moving in spacetime of mass M

then deform metric such that motion agrees with PN & numerical simulations