Tidal interactions of compact binaries

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Newtonian case: modes and overlap for barotropic stars

E.g. Press, Teukolsky, ApJ **213** (1977) 183; Rathore, Broderick, Blandford, MNRAS **339** (2003) 25

$$\ddot{\vec{\xi}} + \mathcal{D}\vec{\xi} = -\vec{\nabla}(\Phi_{\text{ext}} + \vec{x}\cdot\ddot{\vec{z}}), \qquad \mathcal{D}\vec{\xi} = -\vec{\nabla}\left\{\left[\frac{c_s^2}{\rho_0} + 4\pi G \Delta^{-1}\right]\vec{\nabla}\cdot(\rho_0\vec{\xi}\,)\right\}$$

$\mathcal{D}$ is Hermitian w.r.t. measure $dm_0 = \rho_0(r)d^3x$ [Chandrasekhar, ApJ **139** (1964) 664–674]:

$$\mathcal{D}\vec{\xi}^{\text{NM}}_{nlm} = \omega^2_{nl}\vec{\xi}^{\text{NM}}_{nlm}, \qquad \int d^3x\,\rho_0\vec{\xi}^{\text{NM}\,\dagger}_{n'l'm}\vec{\xi}^{\text{NM}}_{nlm} = \delta_{n'n}\delta_{l'l}\delta_{m'm}$$

Allows decompositions of displacement $\vec{\xi}$ in terms of amplitudes $A_{nlm}(t)$:

$$\vec{\xi} = \sum_{nlm} A_{nlm}(t)\vec{\xi}^{\text{NM}}_{nlm}(\vec{x})$$

Amplitudes $A_{nlm}(t)$ satisfy harmonic oscillator equation: $\qquad \rho^{\text{NM}}_{nlm} := -\vec{\nabla}\cdot(\rho_0\vec{\xi}^{\text{NM}}_{nlm})$

$$\ddot{A}_{nlm} + \omega^2_{nl}A_{nlm} = f_{nlm}, \qquad f_{nlm} := -\int d^3x\,\rho^{\text{NM}\,*}_{nlm}(\Phi_{\text{ext}} + \vec{x}\cdot\ddot{\vec{z}})$$

Overlap integrals I_{nl} from multipole decomposition:

$$f_{nlm} \propto I_{nl} \times [l\text{-pole of } \Phi_{\text{ext}}(\vec{z})], \qquad q^{lm} \propto \sum_n I_{nl}A_{nlm}$$

$$I_{nl} \propto \int dr\, r^{l+2}\rho^{\text{NM}}_{nl}(r), \qquad \rho^{\text{NM}}_{nlm} = \rho^{\text{NM}}_{nl}(r)Y^{lm}(\Omega)$$

Effective action for tidal interaction beyond the adiabatic case

$$S_{\text{PP}} = \int d\tau \left[-m - \frac{1}{2} E_{ab} Q^{ab} + ... \right]$$

- Quadrupole $l = 2$ response function F [Goldberger, Rothstein, PRD **73** (2006) 104030]:

$$Q^{ab}(\tau) = -\frac{1}{2} \int d\tau' \; F^{ab}{}_{cd}(\tau, \tau') \, E^{cd}(\tau')$$

- Ansatz:

$$F^{ab}{}_{cd}(\tau, \tau') = F(\tau - \tau') \hat{\delta}^{ab}{}_{cd} \quad \Rightarrow \quad \tilde{Q}^{ab}(\omega) = -\frac{1}{2} \tilde{F}(\omega) \tilde{E}^{ab}(\omega)$$

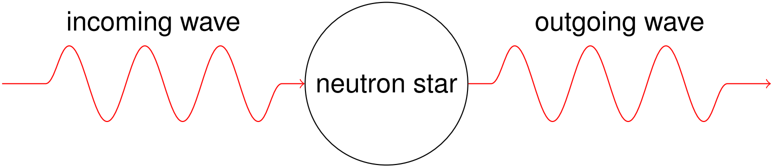
- Connection to Love number: $\tilde{F}(\omega) = 2\mu_2 + i\omega\lambda + 2\omega^2 \mu_2' + \mathcal{O}(\omega^3)$
  - μ_2: 2nd kind relativistic Love number [Hinderer, ApJ **677** (2008) 1216; Damour, Nagar, PRD 80 (2009) 084035; Binnington, Poisson, PRD 80 (2009) 084018]
  - λ: absorption [Goldberger, Rothstein, PRD **73** (2006) 104030]
  - μ_2': beyond adiabatic [Bini, Damour, Faye, PRD **85** (2012) 124034]
- Newtonian case: response is sum of harmonic oscillators

$$\tilde{F}(\omega) = \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

[Chakrabarti, Delsate, Steinhoff, arXiv:1306.5820]

Relativistic response I

[Chakrabarti, Delsate, Steinhoff, arXiv:1304.2228]



- Inhom. Regge-Wheeler eq. with effective δ-source S representing a NS

$$\frac{d^2X}{dr_*^2} + \left[\left(1 - \frac{2M}{r}\right)\frac{l(l+1) - \frac{6M}{r}}{r^2} + \omega^2\right] X = S$$

- Analytic solutions for hom. eq. [Mano, Suzuki, Takasugi, PTP **96** (1996) 549]

$$X_{\mathrm{UV}}^{\nu} = e^{-i\omega r}(\omega r)^{\nu}\left(1 - \frac{2M}{r}\right)^{-i2M\omega} \sum_{n=-\infty}^{\infty} \cdots \times \left[\frac{r}{2M}\right]^{n} {}_2F_1(\dots; 2M/r)$$

$$X_{\mathrm{IR}}^{\nu} = e^{-i\omega r}(\omega r)^{\nu}\left(1 - \frac{2M}{r}\right)^{-i2M\omega} \sum_{n=-\infty}^{\infty} \cdots \times (\omega r)^{n} \, {}_1F_1(\dots; 2i\omega r)$$

- renormalized angular momentum: $\nu = \nu(l, M\omega)$ transcendental number

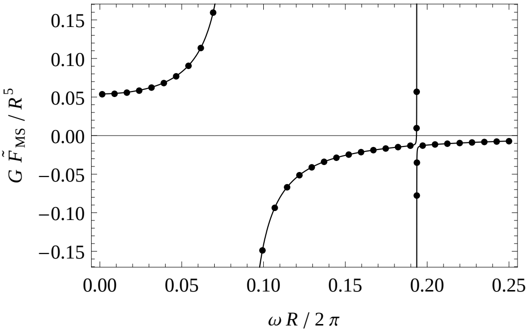
Relativistic response II

[Chakrabarti, Delsate, Steinhoff, arXiv:1304.2228]

- Nonanalytic terms identical $\Rightarrow$ matching: $X_{\rm UV}^{\nu} = K_\nu X_{\rm IR}^{\nu}$
- Sets on indep. solutions: $X_{\rm UV}^{\nu}$, $X_{\rm UV}^{-\nu-1}$ or $X_{\rm IR}^{\nu}$, $X_{\rm IR}^{-\nu-1}$
- From numerical NS perturbation: $X = A_1 X_{\rm UV}^{\nu} + A_2 X_{\rm UV}^{-\nu-1}$
- $X_{\rm IR}^{\nu}$, $X_{\rm IR}^{-\nu-1}$ related to effective δ-source via variation of parameters
- Regularization: $\delta(\vec{r}) = (rc_l)^{2-l} \frac{\Gamma(\frac{d-\epsilon}{2})}{\pi^{3/2} 2^\epsilon \Gamma(\frac{\epsilon}{2})} \mu_0^\epsilon r^{\epsilon-3}$

- Fit for the response:

$$\tilde{F}_{\rm MS}(\omega) = \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$



- Just like in Newtonian case!
- Relative error less than 2%
- Mode frequencies: ω_n
- Relativistic overlap integrals: I_n
- Matching scale μ_0 is fitted, too

Conclusions

Motivation:

- Adiabatic tidal effects may not be sufficient

  [Maselli, Gualtieri, Pannarale, Ferrari, PRD **86** (2012) 044032]
- Resonances between oscillation modes and orbital motion:
  - Shattering of NS crust

    [Tsang, Read, Hinderer, Piro, Bondarescu, PRL **108** (2012) 011102]
  - Numerical simulations of binary NS

    [Gold, Bernuzzi, Thierfelder, Brügmann, Pretorius, PRD **86** (2012) 121501]
- Definition of source (Dixon) multipoles see also [Harte, CQG **29** (2012) 055012]
- Definition of relativistic overlap integrals

Outlook:

- More realistic NS models: rotation, crust, . . . (also for Newtonian case)
- Dimensional regularization
- Other multipoles based on action in [Goldberger, Ross, PRD **81** (2010) 124015]
- 2nd Love number of rotating black holes

Thank you for your attention

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