QFT methods for gravitational wave astronomy

application to spin effects and dynamic tides

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Max-Planck-Institute for Gravitational Physics (Albert-Einstein-Institute), Potsdam-Golm, Germany

Fields and Strings Seminar, Humboldt University Berlin, June 1st, 2016

Outline

Spin and tidal effects

pics/binarywave2

binary black holes

Spin and tidal effects

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binary black holes

GW150914 a.k.a. The Event

peak strain 10^{-21} at $\sim$ 1 Gly
$\Rightarrow$ strain 10^{-7} at 1 AU

3 $M_\odot$ radiated in a fraction of a second
$\Rightarrow$ power $>$ all stars in the visible universe

Spin and tidal effects

pics/ligo

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pics/ligo

LIGO, www.ligo.org

- black holes
  $\rightarrow$ large spin
- strong precession
  $\rightarrow$ tests of gravity
- compute spin effects!

Spin and tidal effects

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pics/neutronstar

www.astroscu.unam.mx/neutrones/

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tidal forces $\leftrightarrow$ oscillation modes
$\Rightarrow$ resonances & dynamic tides!

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$\longrightarrow$ **spin effects**

$\longrightarrow$ **dynamical tides**

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Upcoming and Planned Observatories

Gravitational wave detectors:

pics/et

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Einstein Telescope

LIGO like detectors:

- Virgo (Italy)
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pics/lisa

eLISA space mission

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Radio astronomy:

pics/ska

Square Kilometre Array (SKA)

Common view on analytic description of binaries

matching of zone, see, e.g., Ireland, etal, arXiv:1512.05650

various zones:

- inner zone (IZ)
  around compact objects
- near zone (NZ)
  for the orbit
- far zone (FZ)
  for the waves

in between: buffer zones (BZ)
for the matching

Problematic: different gauges
in different zones

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Effective field theory for compact objects in gravity

Goldberger, Rothstein, PRD **73** (2006) 104029; Goldberger, arXiv:hep-ph/0701129

pics/GRtower

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zones $\rightarrow$ scales

separation of scales:

- scale μ
- object size r_s
- orbital size r
- velocity v
  $\rightarrow$ frequency $\sim \frac{v}{r}$

effective action replaces buffer zones
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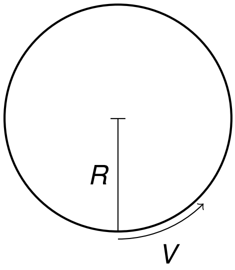
Outline

Two Facts on Spin in Relativity

1. Minimal Extension



- ring of radius R and mass M
- spin: $S = R\,M\,V$
- maximal velocity: $V \leq c$
  $\Rightarrow$ minimal extension:

$$R = \frac{S}{MV} \geq \frac{S}{Mc}$$

2. Center-of-mass

- now moving with velocity v
- relativistic mass changes inhom.
- frame-dependent center-of-mass
- need spin supplementary condition:

$$\text{e.g.,} \quad S^{\mu\nu} p_\nu = 0$$

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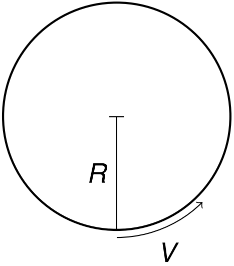

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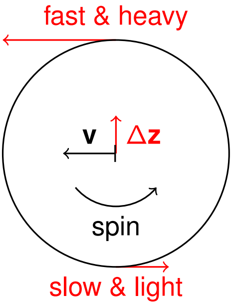

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Point Particle Action in General Relativity

Westpfahl (1969); Bailey, Israel (1975); Porto (2006); Levi & Steinhoff (2014)

$$S_{\rm PP} = \int d\sigma \left[-m\sqrt{g_{\mu\nu}u^\mu u^\nu} + \dots \right]$$

Legendre transformation in u^μ (from Nambu-Goto to Polyakov action)

$$S_{\rm PP} = \int d\sigma \left[p_\mu \frac{Dz^\mu}{d\sigma} - \frac{\lambda}{2}\mathcal{H} + \frac{1}{2} S_{\mu\nu}\Lambda_A{}^\mu \frac{D\Lambda^{A\nu}}{d\sigma} - \frac{p_\mu S^{\mu\nu}}{p_\rho p^\rho}\frac{Dp_\nu}{d\sigma} - \chi^\mu \mathcal{C}_\mu \right]$$

- constraint, interactions in $\mathcal{M}$: $\mathcal{H} := p_\mu p^\mu + \mathcal{M}^2 = 0$ mass-shell constraint
- $\Lambda_A{}^\mu$: frame field on the worldline, "body-fixed" frame

Action is invariant under a "spin gauge symmetry":

- spin gauge constraint: $\mathcal{C}_\mu := S_{\mu\nu}(p^\nu + p\Lambda_0{}^\mu)$ $\sim$ generator of symmetry
- action invariant under a boost of $\Lambda_A{}^\mu$ plus transformation of $S^{\mu\nu}$
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Post-Newtonian Approximation

Application: post-Newtonian approximation

- for bound orbits
- one expansion parameter, $\epsilon_{\mathrm{PN}} \sim \frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \ll 1$ (weak field & slow motion)
- propagation "almost" instantaneous
- time dependence of metric $g_{\mu\nu}$ treated perturbatively

$\rightarrow$ Kaluza-Klein decomposition useful [Kol, Smolkin, arXiv:0712.4116]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij} dx^i dx^j$$

Leads to gravitomagnetic analogy:

- ϕ : gravito-electric field
- A_i : gravito-magnetic field
- γ_{ij} : spin-2 field

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Spin and Gravitomagnetism

Interaction with gravito-magnetic field $A_i \approx -g_{i0}$:

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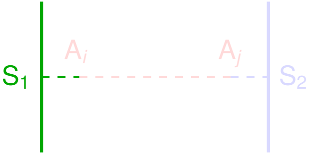

- Here: $r_{12} = |\vec{x}_1 - \vec{x}_2|$
- Ignoring factors like $\delta(t_1 - t_2)$
- Feynman rules see e.g. [arXiv:1501.04956]
- Status: NNLO (many more diagrams, loops)

$$L_{S_1 S_2} = \frac{1}{2} S_1^{ki} \langle \partial_k A_i \, \partial_\ell A_j \rangle \frac{1}{2} S_2^{\ell j}$$

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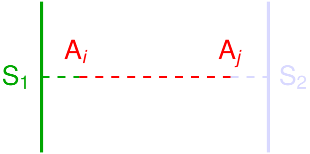

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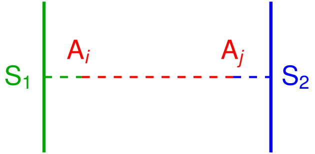

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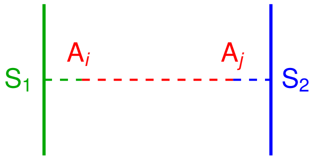

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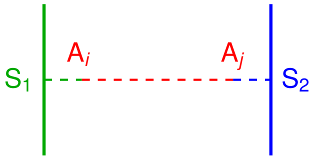

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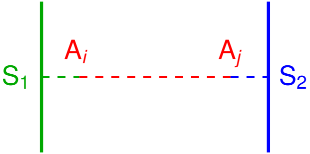

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Results for post-Newtonian approximation with spin

conservative part of the motion of the binary

post-Newtonian (PN) approximation: expansion around $\frac{1}{c} \to 0$ (Newton)

order	c^0 N	c^{-1}	c^{-2} 1PN	c^{-3}	c^{-4} 2PN	c^{-5}	c^{-6} 3PN	c^{-7}	c^{-8} 4PN
non spin	✔		✔		✔		✔		✔
spin-orbit				✔		✔		✔	
S_1^2					✔		✔		✔ (derived last year)
$S_1 S_2$					✔		✔		✔
Spin^3								✔ (derived last year) (✓)	
Spin^4									✔ (derived last year) (✓)
⋮									⋱

✔ known (✓) partial ✔ derived last year

Work by many people ("just" for the spin sector): Barker, Blanchet, Bohé, Buonanno, O'Connell, Damour, D'Eath, Faye, Hartle, Hartung, Hergt, Jaranowski, Marsat, Levi, Ohashi, Owen, Perrodin, Poisson, Porter, Porto, Rothstein, Schäfer, Steinhoff, Tagoshi, Thorne, Tulczyjew, Vaidya

Outline

A neutron star model

pics/neutronstar

Neutron star picture by D. Page
www.astroscu.unam.mx/neutrones/

„Lab“ for various areas in physics

- magnetic field, plasma
- crust (solid state)
- superfluidity
- superconductivity
- unknown matter in core
  condensate of quarks, hyperons, kaons, pions, . . . ?

  accumulation of dark matter ?

Related objects:

- pulsars
- magnetars
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Neutron Star Equations of State

pics/NSMassRad

mpifr-bonn.mpg.de

Neutron Star Equations of State

However, a measurement of the tidal deformations through gravitational waves can be a good alternative to radius measurements.

mpifr-bonn.mpg.de

Dynamical tides of neutron stars

JS etal, in preparation   Hinderer etal, PRL **116** (2016) 181101 (Research Highlight in Nature)

- Neutron stars can oscillate → effective harmonic oscillator action

$$L_{\text{tide}} \sim \frac{DA^{\mu\nu}}{d\sigma}\frac{DA_{\mu\nu}}{d\sigma} - \omega_0^2 A^{\mu\nu}A^{\mu\nu} - \frac{I_0}{2}A^{\mu\nu}E_{\mu\nu} + \frac{K}{4}E^{\mu\nu}E_{\mu\nu} + \ldots$$

(electric) tidal field $E_{\mu\nu} = R_{\alpha\mu\beta\nu}u^\alpha u^\beta$

- Amplitude $A^{\mu\nu}$ must be SO(3) irreducible, i.e, symmetric-tracefree:

$$A^{[\mu\nu]} = 0 = A^{\mu}{}_{\mu}, \qquad A^{\mu\nu}u_\nu = 0$$

- Here: quadrupolar amplitude, strongly couples to gravity
- More oscillators in the action → more modes → spectrum
- In Newtonian limit: easy to compute constants ω_0, I_0, K, ...

How to get these constants in relativistic case? → matching

Other important references:
Hinderer, ApJ 677 (2008) 1216
Alexander, MNRAS 227 (1987) 843
Flanagan, Hinderer, PRD 77 (2008) 021502
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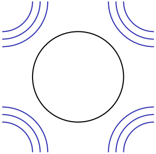
Convenient concept: response function

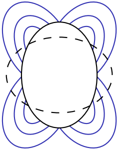
Chakrabarti, Delsate, Steinhoff, PRD **88** (2013) 084038 and arXiv:1304.2228

quadrupolar sector: $\ell = 2$



linear response

$\xrightarrow{F}$



external quadrupolar field $\longrightarrow$ deformation $\longrightarrow$ quadrupolar response

$\phi \sim r^{\ell} \sum {}_2F_1$ $\qquad$ $\phi \sim r^{-\ell-1} \sum {}_2F_1$

quadrupolar response fit:

$$F \approx \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

ω_n: mode frequency
I_n: coupling constant
R: radius

poles $\Rightarrow$ resonances!

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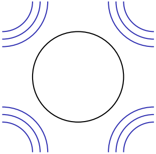

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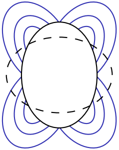

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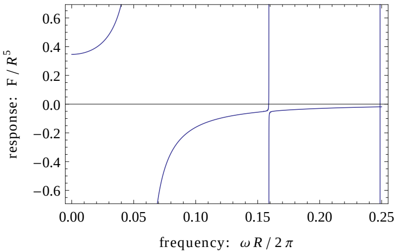

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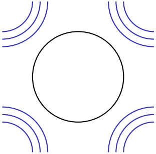

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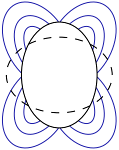

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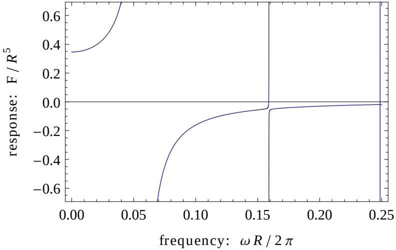

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pics/resonance

Tacoma Bridge

Relativistic effects on dynamic tides

All that was said hold also in Newtonian gravity ...

What is new in the relativistic case?

- values of constants ω_0, I_0, K, ...
  is different
- Newtonian: $K = 0$
  relativistic $K \neq 0$
- redshift effect
- frame dragging effect
  $\sim$ Zeeman effect

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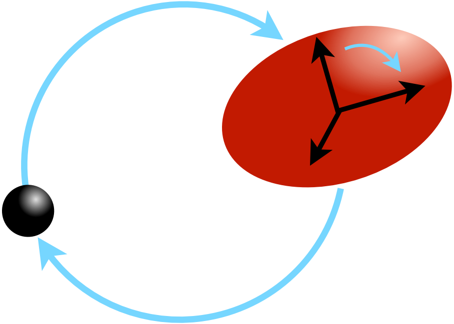

frame of the neutron star is dragged in the direction of the orbital motion

Summary

- black holes $\rightarrow$ large spins
  $\rightarrow$ strong gravitomagnetic effects
  $\rightarrow$ interesting/new tests of gravity
- NS structure largely unknown/mysterious
  $\rightarrow$ tidal effects in gravitational waves and electromagnetic counterparts can enlighten nuclear interactions

GW astronomy exciting new and interdisciplinary field

- important to use common language $\rightarrow$ effective field theory
- but need to go a bit beyond standard techniques
  e.g., in-in formalism, see [Galley PRL 110, 174301 (2013)]
- "good" level of abstraction:

  DeWitt, The Global Approach to Quantum Field Theory