Oscillation modes, tides, and resonances

of compact objects (2nd part)

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Newtonian case

- Amplitudes:

$$\ddot{A}_{nlm} + \omega_{nl}^2 A_{nlm} = f_{nlm}$$

- Overlap integrals I_{nl}:

$$f_{nlm} = -\frac{1}{l!} I_{nl} \Phi_{\text{ext}}^{lm}, \qquad q^{lm} = \sum_n I_{nl} A_{nlm}$$

- Linear response in frequency domain:

$$\tilde{q}^{lm}(\omega) = -\frac{1}{l!} \tilde{F}_l(\omega) \tilde{\Phi}_{\text{ext}}^{lm}(\omega)$$

- Newtonian case: response is sum of harmonic oscillators

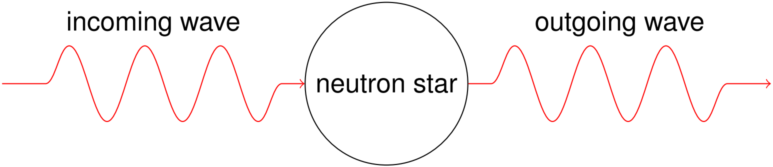
$$\tilde{F}_l(\omega) = \sum_n \frac{I_{nl}^2}{\omega_n^2 - \omega^2}$$

[Chakrabarti, Delsate, Steinhoff, arXiv:1306.5820]

- From now on: $l = 2$

Relativistic response I

[Chakrabarti, Delsate, Steinhoff, arXiv:1304.2228]



- Inhom. Regge-Wheeler eq. with effective δ-source S representing a NS

$$\frac{d^2 X}{dr_*^2} + \left[\left(1 - \frac{2M}{r}\right) \frac{l(l+1) - \frac{6M}{r}}{r^2} + \omega^2 \right] X = S$$

- Analytic solutions for hom. eq. [Mano, Suzuki, Takasugi, PTP **96** (1996) 549]

$$X^\nu_{\mathrm{UV}} = e^{-i\omega r}(\omega r)^\nu \left(1 - \frac{2M}{r}\right)^{-i2M\omega} \sum_{n=-\infty}^{\infty} \cdots \times \left[\frac{r}{2M}\right]^n {}_2F_1(\ldots; 2M/r)$$

$$X^\nu_{\mathrm{IR}} = e^{-i\omega r}(\omega r)^\nu \left(1 - \frac{2M}{r}\right)^{-i2M\omega} \sum_{n=-\infty}^{\infty} \cdots \times (\omega r)^n \, {}_1F_1(\ldots; 2i\omega r)$$

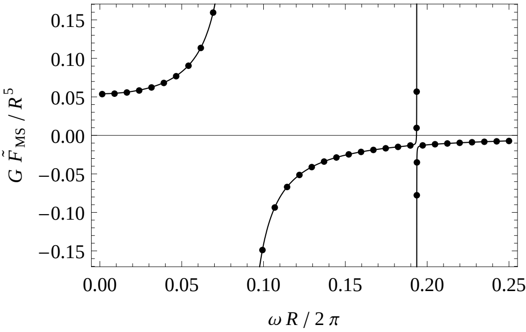
- renormalized angular momentum: $\nu = \nu(l, M\omega)$ transcendental number

- Nonanalytic terms identical $\Rightarrow$ matching: $X^{\nu}_{\text{UV}} = K_{\nu} X^{\nu}_{\text{IR}}$
- Sets on indep. solutions: $X^{\nu}_{\text{UV}}, X^{-\nu-1}_{\text{UV}}$ or $X^{\nu}_{\text{IR}}, X^{-\nu-1}_{\text{IR}}$
- From numerical NS perturbation: $X = A_1 X^{\nu}_{\text{UV}} + A_2 X^{-\nu-1}_{\text{UV}}$
- $X^{\nu}_{\text{IR}}, X^{-\nu-1}_{\text{IR}}$ related to effective δ-source via variation of parameters
- Regularization: $\delta(\vec{r}) = (rc_l)^{2-l} \frac{\Gamma(\frac{d-\epsilon}{2})}{\pi^{3/2} 2^{\epsilon} \Gamma(\frac{\epsilon}{2})} \mu_0^{\epsilon} r^{\epsilon - 3}$

- Fit for the response:

$$\tilde{F}_{\text{MS}}(\omega) = \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

- Just like in Newtonian case!
- Relative error less than 2%
- Mode frequencies: ω_n
- Relativistic overlap integrals: I_n
- Matching scale μ_0 is fitted, too



Conclusions

Motivation:

- Adiabatic tidal effects may not be sufficient
  [Maselli, Gualtieri, Pannarale, Ferrari, PRD **86** (2012) 044032]
- Resonances between oscillation modes and orbital motion:
  - Shattering of NS crust
    [Tsang, Read, Hinderer, Piro, Bondarescu, PRL **108** (2012) 011102]
  - Numerical simulations of binary NS
    [Gold, Bernuzzi, Thierfelder, Brügmann, Pretorius, PRD **86** (2012) 121501]
- Definition of source (Dixon) multipoles see also [Harte, CQG **29** (2012) 055012]
- Definition of relativistic overlap integrals

Outlook:

- More realistic NS models: rotation, crust, ... (also for Newtonian case)
- Dimensional regularization
- Other multipoles based on action in [Goldberger, Ross, PRD **81** (2010) 124015]
- 2nd Love number of rotating black holes

Thank you for your attention

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